A Control Framework for Interactive Deformable Image Registration

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Abstract. A novel framework for interactive image registration is proposed. In this formulation, a closed loop system is created to control the PDE defining the registration evolution; a human expert user is present “in the loop” coupled with a convenient choice of automatic algorithm. The result is a system that leverages the expert’s high level anatomical knowledge with an automatic method’s speed in performing tedious correction tasks.

Keywords: deformable registration, interactive, PDE, control, Bspline

1 Introduction

Medical imaging applications that require multiple imaging sessions over the course of the treatment, make comparisons between patients, or fuse information from several imaging modalities contain a registration component. A number of automatic methods have been proposed \cite{9, 5, 3}, but they are adversely affected by the presence of artifacts, large misalignments, effects of disease/trauma, and population variance.

A parametric model for free-form deformations was proposed by Rueckert \textit{et.al} \cite{6}; B-splines deformations are defined by uniformly placed anchor points within the image domain. The optimal solution is computed via gradient descent to minimize an energy composed of an image similarity and a regularization term, which controls the smoothness of the deformation. When the registration fails using B-splines, it is often due to one of the following reasons: an improper choice of the regularization weight (unknown ahead of time), the algorithm becoming trapped in a local minimum, or the similarity metrics inability to discriminate in a certain region.

Another class of models to define a deformation is the physical model. An example is the \textit{Demons} \cite{8} algorithm in which registration is phrased as a diffusion process in which the image diffuses through semi-permeable membranes.
Most commonly, in practice, an effector is located at each pixel and the moving image is deformed according to the optical flow computed between the static and moving images. In [2], Christensen et. al introduced a viscous flow model that avoids penalizing large deformations by choosing an appropriate regularizer. A state-of-the-art method called SyN [1] extends this theory by formulating the problem symmetrically and computing a geodesic on the manifold of diffeomorphic mappings relating the images. Despite their strong performance on certain classes of images, incorrect registrations are computed in scenarios where a few salient features dwarf the contribution to the cost functional of the background, which has poor contrast in comparison.

To mitigate the drawbacks of automatic methods, a number of interactive registration methods have been proposed. For instance, in [4], the authors query the user for several corresponding landmarks in the images that are used as soft constraints during the registration. Balancing the image term with the penalty for misalignment of landmarks is a delicate task, the number of landmarks needed is unknown ahead of time, and the locations in which they should be placed is also unknown. Furthermore, the penalty for each landmark is uniform, and the effect is dependent on the spacing of the B-spline anchor points. A related work [7] defines the deformation vector field by radial basis functions (RBF) and minimizes an energy that balances the matching of landmarks with the smoothness of the deformation. In this work, image intensities are not considered, and the same difficulties mentioned above exist.

**Contribution:** This work presents a principled framework for designing an interactive registration algorithm. It considers existing, automatic algorithms as open loop systems and describes how to design a closed loop control system around them. The system includes a “user in the loop”: sparse interactions from the user guide the algorithm globally and finer misregistration errors are corrected by the automatic method. As a result, the framework seamlessly leverages the user’s global, anatomical knowledge with an automatic algorithm’s ability to correct local deformations, an otherwise tedious task for the user. Advantages of this approach include the ability to provide online input only in problematic areas (reducing user effort), a natural way to use spatially varying penalties for misalignment, and a clear approach for incorporating prior information.

## 2 Methods

### 2.1 Problem Analysis

We denote the deformation field as $\phi(x,t)$; it is the solution to a certain, convenient partial differential equation (PDE). In this work, a class of registration problems that can be solved by finding the solution of a PDE of the form

$$\phi_t = G(\phi, I_s, I_m)$$  \hspace{1cm} (1)

is considered. Usually, to select the function $G(\cdot)$, an energy $E(\phi, I_s, I_m)$ is defined; it is a measure of similarity between a stationary image $I_s$ and a moving
image $I_m$, warped according to $\phi$. The goal is to minimize $E(\cdot)$ in which case the function $G(\cdot)$ should be chosen to decrease $E(\cdot)$ as rapidly as possible.

One formulation that falls into this class of problems is the B-spline deformable registration method proposed in [6]. The particular energy chosen takes the following form:

$$E(\phi, I_s, I_m) = -E_{sim}(I_s, T(I_m, \phi)) + \lambda E_{smooth}(T(\phi))$$  \hspace{1cm} (2)

$$E_{sim}(I_0, I_1) = H(I_0) + H(I_1) - H(I_0, I_1) ,$$  \hspace{1cm} (3)

$$E_{smooth}(T) = \int_0^x \int_0^y \int_0^z \left[ \left( \frac{\partial^2 T}{\partial x^2} \right)^2 + \left( \frac{\partial^2 T}{\partial y^2} \right)^2 \left( \frac{\partial^2 T}{\partial z^2} \right)^2 + 2 \left( \frac{\partial^2 T}{\partial x \partial y} \right)^2 \right] .$$  \hspace{1cm} (4)

In Eq. 3, $H(\cdot)$ and $H(\cdot, \cdot)$ are the marginal and joint entropies, respectively, and $T(\cdot)$ is the B-spline deformation. $T(\cdot)$ is completely defined by the vector of control points $\phi$, which are the parameters of the B-spline deformation. Consequently, decreasing $E(\cdot)$ most rapidly corresponds to evolving the parameters according to the gradient and choosing $G(\cdot)$ as:

$$\phi_t = G(\phi, I_s, I_m) = \frac{\partial E(\phi, I_s, I_m)}{\partial \phi} .$$  \hspace{1cm} (5)

Eq. 1 defines an open loop systems whose critical points are determined purely by the nominal dynamics $G(\cdot)$, derived from a chosen $E(\cdot)$. It is well known that the limitation of this type of automatic algorithm lies in the difficulty designing a “good” energy (i.e., one that has a minimum corresponding to the correct registration). The solution proposed in this work is to make a closed loop control system around Eq. 1; the details are presented in Section 2.2. Closing the control loop has the effect that the dynamics of $\phi$ now contain an additional control term:

$$\phi_t = G(\phi, I_m, I_s) + H(\xi) .$$  \hspace{1cm} (6)

Here, $\xi$ is defined as

$$\xi = \psi - \phi$$  \hspace{1cm} (7)

where $\xi$ is an estimate for the ground truth deformation relating the images. A detailed explanation about the nature of $\xi$ is found in Section 2.3.

2.2 Control Framework

The diagram in Fig. 1 contains a visual description of the registration framework discussed in subsequent sections. When the registration begins, no user input has been provided, and the deformation $\phi$ as well as its estimate $\psi$ are initialized. An
Fig. 1. The interactive control framework.

The user, through the controller, generates the input signal $u_k$ to evolve the automatic algorithm, the green box, for a predefined time after which it is paused and the current registration result is visualized for the user. It is assumed that the user knows the ground truth registration $\phi^*$ and the process continues until the quantity $\|\phi - \phi^*\|$ falls below the desired threshold. Based on the discrepancy between the current registration and the ground truth, the user has the opportunity to provide input $u_k$; this input is accumulated over time into $U$ and guides the estimate $\psi$ at the next stage. Finally, the loop is closed by computing the error $\xi$ between the estimate and the current deformation. This error influences the dynamics $\phi_t$ to bias the registration according to the user’s input.

In Fig. 1, the control signals $u_k$ at each discrete time point $k$ are generated by the expert user in the control loop. While a number of different interactions are possible within the proposed framework, for the discussion that follows, $u_k$ is a position vector paired with a direction vector, respectively: $x_k, v_k$. These quantities define the desired perturbation of the deformation field, as marked by the user. An example of user input is shown in Fig. 2 as well as its effect on the dynamics of the system.

The user input as a function of time is defined as the following sum:

$$ h(x, t) = \sum_{i=0}^{k} v_i h_0(x - x_i, v_i) \delta(t - t_i) \text{ where } (8) $$

$$ h_0(x - x_n, v_n) = e^{-\frac{1}{2\pi \sigma_n^2} (x - x_n)^T (x - x_n)} \text{(9)} $$

and $\delta(\cdot)$ is the Dirac delta function. Thus, the cumulative input provided by the expert up to time $t_k$ is defined as

$$ U(x, t) = \int_0^t h(x, t) dt \text{ .} \text{(10)} $$

This input will be used to compute the estimate of the ground truth deformation in Section 2.3.
The ground truth \( \phi^* \) is the user’s idea for the correct alignment of two images. It is hidden from the registration algorithm and only hints about its structure become available over time as the user monitors the registration, occasionally providing guidance. The guidance is sparse spatially and temporally. The goal of the estimator is to infer \( \phi^* \) from the input; this inference is denoted by \( \psi \).

In general, the estimator is a function of the current deformation field \( \phi \), the cumulative user input \( U \), and any prior information available about the registration problem \( \theta_{prior} \) (e.g., landmarks, corresponding intensities, shape models, etc.). This function can be written as:

\[
\psi = F(U, \phi, \theta_{prior}) .
\] (11)

A simple choice for \( F(\cdot) \), in the absence of any prior information about the problem at hand is

\[
F(U, \phi, \theta_{prior}) = \phi + U .
\] (12)

The effect of this choice is to bias the dynamics of \( \phi \) for the consecutive evolution. Furthermore, this selection of \( F(\cdot) \) reduces Eq. 6 to

\[
\phi_t = G(\phi, I_m, I_s) + H(U) .
\] (13)

Here, \( H(\cdot) \) is a function that ensures stability of the system. The results in Section 3 are computed using this simple choice of estimator and the B-spline method for the registration algorithm (green box in Fig. 1).
Fig. 3. A synthetic experiment is presented in which a deformation Fig. 3(a) is recovered using the classical B-spline algorithm Fig. 3(b)-3(c) and the proposed control framework with the chosen automatic registration algorithm being B-splines Fig. 3(d)-3(e).

3 Results

Two experiments were performed to illustrate the effect of using closed loop control for registration. The first, shown in Fig. 3, is a qualitative experiment in which a phantom image is warped according to a known, ground truth deformation. The ground truth deformation is the red grid and the computed deformation is shown in blue. The two images being aligned are illustrated by placing the static image in the green color channel and the moving image in the red channel. Since the control framework in this example is closing the loop around the B-spline registration method, a comparison with the classical, open loop B-spline formulation [6] is shown. As can be seen in Fig. 3(c) 3(d), the interactive framework achieves excellent results even for a large initial misalignment while open loop B-spline registration leaves large errors.

In Fig. 4, a second experiment is shown. There, a human brain with a meningioma present is warped; again, it is assumed the ground truth deformation is available so that a quantitative comparison can be made. In this experiment, a user is simulated for the control framework by providing input in areas where
large misalignment is present. An error plot comparing the control framework as to classical B-splines is shown in Fig. 4(d). Registration is stopped after twenty iterations, which is a simulated threshold for the user’s available time; in theory, the fine, remaining misalignments seen in Fig. 4(e)-4(f) can be corrected and the deformation can be driven further towards the ground truth at the cost of increased user effort.

The experiments presented in this section demonstrate the advantage of “closing the loop” around an open-loop, semi-automatic method, such as B-spline registration. With sufficient input, the function $H(\cdot)$ can be made large enough to overtake the contribution of the data driven term $G(\cdot)$ in Eq. 13, in case the solution found by the automatic method is different from the user’s expert knowledge. This property prevents the deformation field from “snapping back” to the incorrect solution; effectively, the energy functional from which the dynamics equation Eq. (5) was derived is changing adaptively, in accordance to the user input. Thus, an added feature of the control framework is that parameter selection is not needed. This approach is cardinally different from landmark based registration. The user waits until the registration completes to add input;
this end result may require significantly more input to correct than is needed if
guidance is provided preemptively, before large errors appear. Also, new land-
marks are likely to change the energy minimum unpredictably, affecting regions
where registration was previously correct. Consequently, user work is expected
to be higher than the interactive approach proposed in this note.

4 Conclusion

In this work, we propose a closed loop control formulation for performing in-
teractive registration. This framework allows for a number of design choices;
in particular, prior information can be incorporated in the estimator, the au-
tomatic algorithm being controlled is interchangeable, the control function can
be designed, and the user interactions can change. For the purpose of having
a concrete discussion, an example using B-splines is presented. Qualitative and
quantitative comparisons are shown to illustrate the improvement on the results
achieved using an open loop system when a closed loop system is employed.
Future work includes investigating various choice of estimators and user inter-
actions, comparing results when other automatic algorithms are placed into the
control loop, and testing on 3D data.

5 Acknowledgments

This research was supported by the National Center for Research Resources un-
der Grant P41-RR-013218, the National Institute of Biomedical Imaging and
Bioengineering under Grant P41-EB-015902 of the National Institutes of Health
through the Neuroanalysis Center of Brigham and Women's Hospital, and the
National Alliance for Medical Image Computing, funded by the National Insti-
tutes of Health through the NIH Roadmap for Medical Research, under Grant
U54 EB005149.

References

analysis of structural and diffusion imaging in traumatic brain injury. Academic
Radiology 15(11), 1360 – 1375 (2008)
2. Christensen, G., Rabbitt, R., Miller, M.: Deformable templates using large defor-
mation kinematics. Image Processing (TIP), IEEE Transactions on 5(10), 1435 –
1447 (1996)
Processing (TIP), IEEE Transactions on 12(11), 1427–1442 (Nov 2003),
http://dx.doi.org/10.1109/TIP.2003.813139
5. Pluim, J.P.W., Maintz, J.B.A., Viergever, M.A.: Mutual-information-based regis-
tration of medical images: a survey. Medical Imaging (TMI), IEEE Transactions on
22(8), 986 – 1004 (2003)

